APPLECROSS
SENIOR HIGH SCHOOL

STUDENT NAME: _____

All working must be shown in the space provided. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than 2 marks, valid working or justification is required to receive full marks.

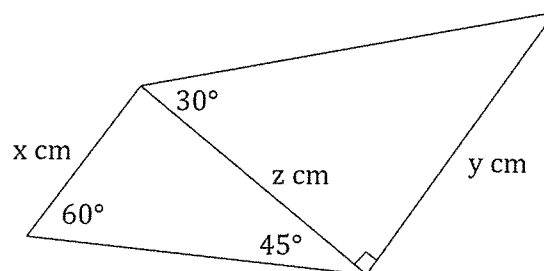|	Total	Result	
Section 1	18		**%**
Section 2	37		
Total	55		

Section 1: Resource – Free

Working time: 19 minutes

Question 1   [3, 2 = 5 marks]
Consider the situation below.
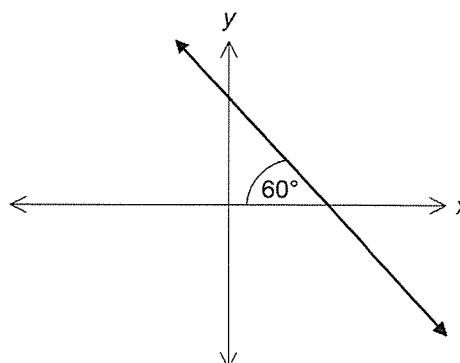
(a)   Use Exact Values and the Sine Rule to determine an expression for the side labelled z cm in terms of x.

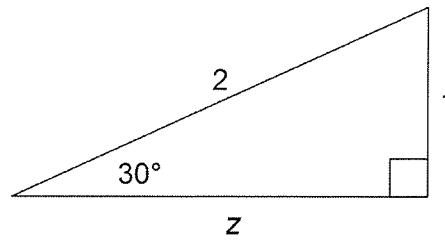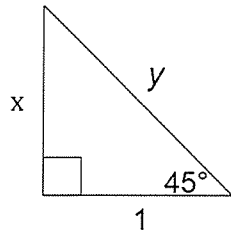(b)   Use the right triangle shown above and trigonometric ratios to show that $y = \frac{x}{\sqrt{2}}$.

Question 2   [2 marks]
Determine the gradient of the drawn line below.
All values should be expressed in exact form.

Question 3   [3, 6, 2, = 11 marks]
Consider the two right triangles shown below.

(a)   Calculate the value of x, y and z.

Now use the triangles above to help you determine the **exact** value of the following. Rationalise denominators where necessary.

(b)   (i) $\sin^2 45° + \cos^2 45°$ (ii) $\tan 30° + \tan 60°$

           (iii) $\cos 150°$ (iv) $\sin 225°$

(c) θ, where $\cos \theta = \frac{\sqrt{3}}{2}$ for $-180° \leq \theta \leq 180°$

END OF SECTION 1

APPLECROSS
SENIOR HIGH SCHOOL

STUDENT NAME: _____

37

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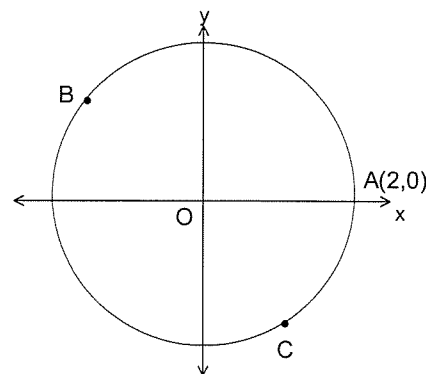
Section 2: Resource – Rich
Working time: 39 minutes

To be provided by the student:
ClassPad and/or Scientific Calculators
1 sheet of A4–sized paper of notes, double-sided

Question 4   [4, 3 = 7 marks]
In the diagram to the right, a circle of radius 2 units is centred at the origin. The major arc ABC is 10 units long.

(a)   What is the size of the acute angle AOC in radians?

(b)   What are the coordinates of point C, correct to two decimal places?
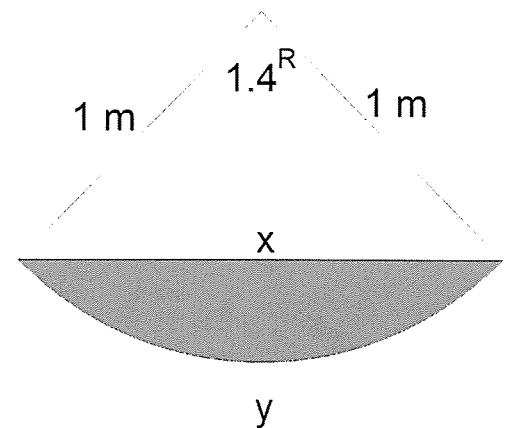
Question 5   [4 marks]

Express sin θ in terms of θ, if, in the diagram below the area of the segment (shaded) is equal to one fifth of the area of the circle.
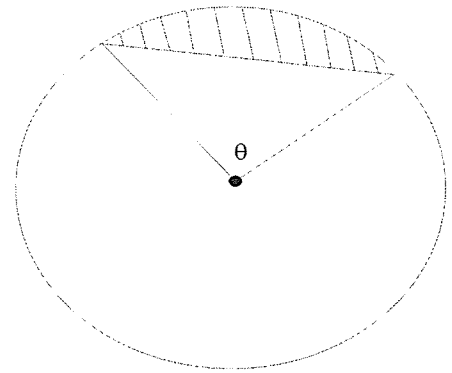
Question 6   [4 marks]

The diagram below represents the cross-section of a water trough. Find the perimeter of the cross-section and the area of the segment drawn.

Question 7   [2 marks]

Through what angle in degrees does a pendulum of length 55 cm swing through, if the arc length traversed by its tip is 16.2 cm? Give your answer correct to two decimal places.

Question 8   [4 marks]

Find the area of a parallelogram with side lengths of 10 cm and 13 cm and including an angle of 30°.

Question 9   [6 marks]

A yacht sails 8 km on a bearing of 070° followed by 10 km on a bearing of 120°. Calculate the bearing needed for the yacht to return directly to it's starting point, correct to two decimal places. [Hint: draw the diagram carefully]

Question 10   [4 marks]
A triangle has an area of 33.3 cm². If two sides of the triangle measure 7.5 cm and 9.2 cm, find the angle(s) size, correct to one decimal place, determined by the two known sides.

Question 11   [6 marks]
Point A represents the top of a pole of height 4 m at an angle of elevation of 60° from B. The distance from B to a point C, further along is 5m. Points B, C and the bottom of the pole are collinear. Calculate the angle of elevation of A from C.

END OF TEST

APPLECROSS
SENIOR HIGH SCHOOL

STUDENT NAME: _____ *Solutions*

	Total	Result	
Section 1	18		
Section 2	37		**%**
Total	55		

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Section 1: Resource – Free

Working time: 19 minutes

Question 1   **[3, 2 = 5 marks]**
Consider the situation below.
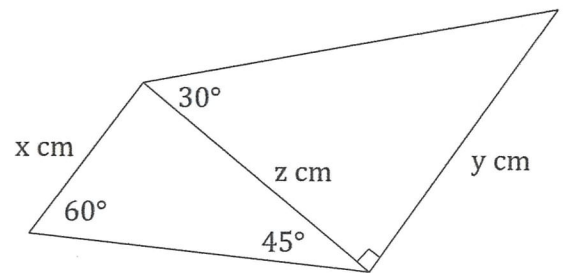
(a)   Use Exact Values and the Sine Rule to determine an expression for the side labelled z cm. (in terms of x)

$$\frac{z}{\sin 60°} = \frac{x}{\sin 45°} \quad \checkmark$$

$$z = \frac{x \cdot \sin 60°}{\sin 45°}$$

$$= \frac{x \cdot \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} \quad \checkmark$$

$$= \frac{x\sqrt{6}}{2} \quad \checkmark$$

(b)   Use the right triangle shown above and trigonometric ratios to show that $y = \frac{x}{\sqrt{2}}$.

$$\tan 30° = \frac{y}{z}$$

$$\frac{1}{\sqrt{3}} = \frac{y}{\frac{x\sqrt{6}}{2}} \quad \checkmark$$

$$y = \frac{x\sqrt{6}}{2} \cdot \frac{1}{\sqrt{3}}$$

$$y = \frac{x \cdot \sqrt{2}\sqrt{3}}{2\sqrt{3}}$$

$$y = \frac{x\sqrt{2}}{2}$$

$$y = \frac{x\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{x \cdot 2}{2\sqrt{2}}$$
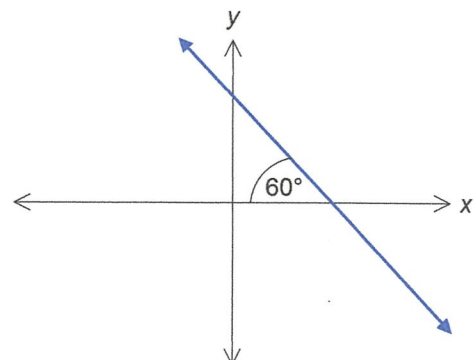
$$= \frac{x}{\sqrt{2}} \quad \checkmark$$

Question 2   **[2 marks]**
Determine the gradient of the drawn line below.
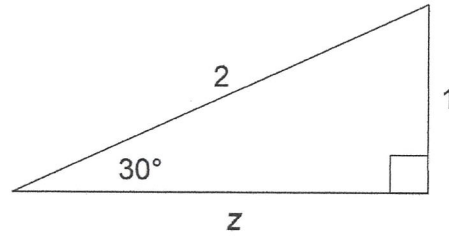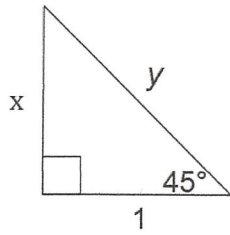All values should be expressed in exact form.

$$M = \tan 120°$$

$$= -\tan 60° \quad \checkmark$$

$$= -\sqrt{3} \quad \checkmark$$

Question 3   [3, 6, 2, = 11 marks]

Consider the two right triangles shown below.

(a)   Calculate the value of x, y and z.

$$x = 1 \quad , \quad y = \sqrt{2} \quad , \quad z = \sqrt{3}$$ ✓✓✓

Now use the triangles above to help you determine the **exact** value of the following. Rationalise denominators where necessary.

(b)   (i) $\sin^2 45° + \cos^2 45°$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$ ✓

(ii) $\tan 30° + \tan 60°$

$$= \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{1}$$

$$= \frac{1 + \sqrt{3} \cdot \sqrt{3}}{\sqrt{3}}$$

$$= \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{3}$$ ✓

(iii) $\cos 150°$

$$= - \cos 30° ✓$$

$$= - \frac{\sqrt{3}}{2}$$ ✓

(iv) $\sin 225°$

$$= - \sin 45° ✓$$

$$= - \frac{1}{\sqrt{2}}$$

$$= - \frac{\sqrt{2}}{2}$$ ✓

(c) θ, where $\cos \theta = \frac{\sqrt{3}}{2}$ for $-180° \le \theta \le 180°$

ref angle $\theta = 30°$

$\cos\theta$ is +ve in $1^{st} + 4^{th}$ Quadrants.

$$\therefore \theta = -30° , 30°$$ ✓ ✓

END OF SECTION 1

Page 2

APPLECROSS
SENIOR HIGH SCHOOL

STUDENT NAME: _Solutions_

37

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Section 2: Resource – Rich
Working time: 39 minutes

To be provided by the student:
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1 sheet of A4–sized paper of notes, double-sided

Question 4   [4, 3 = 7 marks]
In the diagram to the right, a circle of radius 2 units is centred at the origin. The major arc ABC is 10 units long.

(a)   What is the size of the acute angle AOC? _in radians._

$$\ell = 2\pi r - 10 \quad \checkmark$$
$$= 2\pi \cdot 2 - 10$$
$$= 4\pi - 10 \quad \checkmark$$
$$\ell = r \cdot \theta$$
$$4\pi - 10 = 2 \cdot \theta \quad \checkmark$$
$$\theta = \frac{4\pi - 10}{2}$$
$$\theta = 2\pi - 5 \quad \checkmark$$
$$\approx 1.28^R$$
$$\approx 73.52°$$

(b)   What are the coordinates of point C, correct to two decimal places?
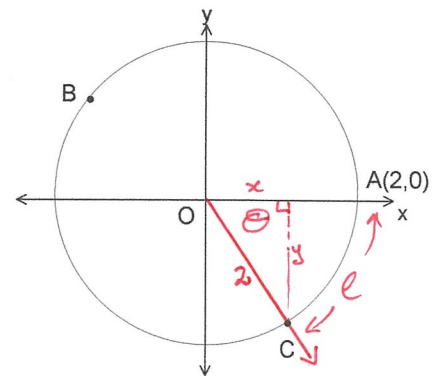
$$\frac{x}{2} = \cos\theta \qquad\qquad y = -\sin\theta$$
$$x = 2\cos(2\pi - 5) \qquad \frac{y}{2} = -2\sin(2\pi - 5)$$
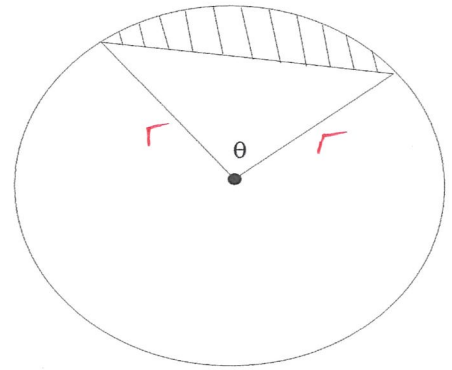$$\approx 0.57 \quad \checkmark \qquad\qquad \approx -1.92 \quad \checkmark$$

$$\therefore \text{ Coordinates of } C; \ (0.57, -1.92)$$
$$\checkmark$$

Question 5   [4 marks]
Express sin θ in terms of θ, if, in the diagram below the area of the segment (shaded) is equal to one fifth of the area of the circle.

$$A(\text{circle}) = \pi r^2$$

$$A(\text{segment}) = \tfrac{1}{2} r^2 (\theta - \sin\theta)$$

$$\tfrac{1}{2} r^2 (\theta - \sin\theta) = \tfrac{1}{5}\pi r^2 \quad\checkmark\checkmark$$

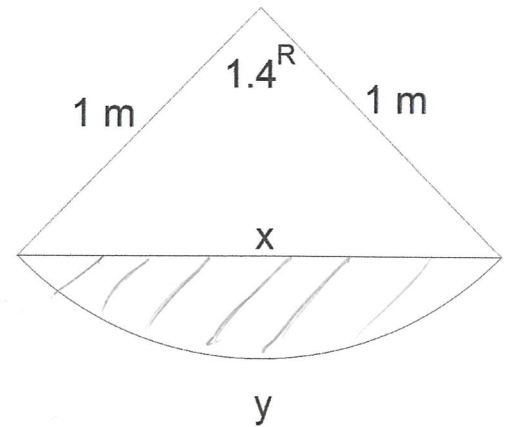$$\frac{\theta}{2} - \frac{\sin\theta}{2} = \frac{\pi}{5}$$

$$\frac{\sin\theta}{2} = \frac{\theta}{2} - \frac{\pi}{5} \quad\checkmark$$

$$\sin\theta = \frac{2\theta}{2} - \frac{2\pi}{5}$$

$$\sin\theta = \frac{5\theta - 2\pi}{5} \quad\checkmark$$

Question 6   [4 marks]
The diagram below represents the cross-section of a water trough. Find the perimeter of the cross-section and the area of the segment drawn.

$$P = x + y$$

$$= \sqrt{1^2 + 1^2 - 2 \cdot 1 \cdot 1 \cdot \cos 1.4^R} + 1 \cdot 1.4^R \quad\checkmark$$

$$\approx \sqrt{1.66} + 1.4$$

$$\approx 1.2884 + 1.4 \quad\checkmark$$

$$= 2.688 \text{ m}$$

$$A = \tfrac{1}{2} \cdot 1^2 (1.4 - \sin 1.4) \quad\checkmark$$

$$= 0.207 \text{ m}^2 \quad\checkmark$$

Question 7   [2 marks]
Through what angle in degrees does a pendulum of length 55 cm swing through, if the arc length traversed by its tip is 16.2 cm? Give your answer correct to two decimal places.

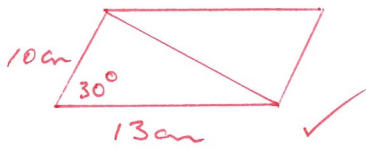$$\ell = r.\theta$$

$$16.2 = 55 . \theta \quad\checkmark$$

$$\theta = \frac{16.2}{55} . \frac{180°}{\pi}$$

$$\approx 0.29 . \frac{180}{\pi}$$

$$\approx 16.88° \quad\checkmark$$

Question 8   [4 marks]

Find the area of a parallelogram with side lengths of 10 cm and 13 cm and including an angle of 30°.

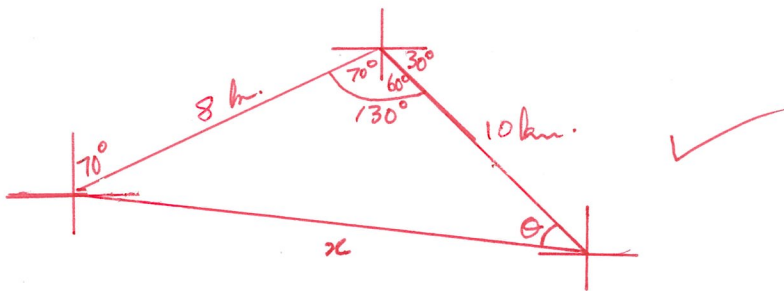$$A(\text{parallelogram}) = 2 \times \left[\frac{1}{2} \cdot 10 \cdot 13 \cdot \sin 30° \right] cm^2$$

$$= 65 \, cm^2$$

Question 9   [6 marks]

A yacht sails 8 km on a bearing of 070° followed by 10 km on a bearing of 120°. Calculate the bearing needed for the yacht to return directly to it's starting point. [Hint: draw the diagram carefully] * correct to 2d.p.

$$x^2 = 8^2 + 10^2 - 2 \cdot 8 \cdot 10 \cdot \cos 130°$$

$$\approx 266 \cdot 846$$

$$x \approx 16 \cdot 3354 \, km$$

$$\frac{\sin \theta}{8} = \frac{\sin 130°}{16 \cdot 3354}$$

$$\sin \theta \approx 0 \cdot 375157$$

$$\theta \approx 22 \cdot 03°$$

∴ Bearing back to starting point is $270° + [30° - 22 \cdot 03°]$

$$= 277 \cdot 97° \, T$$

Question 10 [4 marks]

A triangle has an area of 33.3 cm². If two sides of the triangle measure 7.5 cm and 9.2 cm, find the angle(s) size determined by the two known sides. ✱ correct to one decimal places.

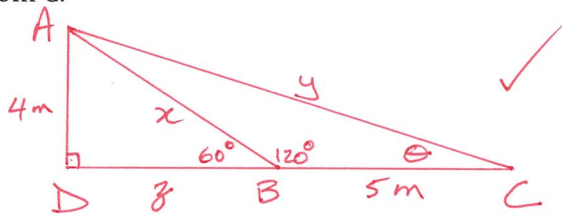$$33.3 = \frac{1}{2} \cdot 7.5 \cdot 9.2 \cdot \sin\theta \quad ✓$$

$$\sin\theta \approx 0.9652 \quad ✓$$

$$\theta \approx 74.8° \text{ or } 105.2°$$
✓   ✓

Question 11 [6 marks]

Point A represents the top of a pole of height 4 m at an angle of elevation of 60° from B. The distance from B to a point C further along is 5m. Points B, C and the bottom of the pole are collinear. Calculate the angle of elevation of A from C.

$$\sin 60° = \frac{4}{x} \quad ✓$$

$$x = \frac{4}{\sin 60°}$$

$$x = \frac{4}{\frac{\sqrt{3}}{2}}$$

$$x = \frac{8}{\sqrt{3}} \quad ✓ \approx 4.6188.$$

or $\triangle ADB$: $\tan 60° = \dfrac{4}{z}$

$$z = \frac{4}{\tan 60°}$$

$$\approx 2.309401077$$

$$\therefore DC = 7.309.\cdots$$

$\triangle ADC$: $\tan\theta = \dfrac{4}{7.309\cdots}$

$$\theta \approx 28.69°$$

$$y^2 = \left(\frac{8}{\sqrt{3}}\right)^2 + 5^2 - 2\left(\frac{8}{\sqrt{3}}\right) \cdot 5 \cdot \cos 120°$$

$$y^2 \approx 69.4273$$

$$y \approx 8.33 \text{ m} \quad ✓$$

$$\frac{\sin 120°}{y} = \frac{\sin\theta}{x} \quad ✓$$

$$\sin\theta = \frac{\frac{8}{\sqrt{3}} \cdot \sin 120°}{8.33}$$

$$\approx 0.480059$$

$$\theta \approx 28.69° \quad ✓$$

∴ The angle of elevation of A from C is 28.69°

END OF TEST